Fluctuation-Driven First-Order Transition in Pauli-limited d-wave Superconductors

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We study the phase transition between the normal and non-uniform (Fulde-Ferrell-Larkin-Ovchinnikov) superconducting state in quasi two-dimensional d-wave superconductors at finite temperature. We obtain an appropriate Ginzburg-Landau theory for this transition, in which the fluctuation spectrum of the order parameter has a set of minima at non-zero momenta. The momentum shell renormalization group procedure combined with ε expansion is then applied to analyze the phase structure of the theory. We find that all fixed points have more than one relevant directions, indicating the transition is of the fluctuation-driven first order type for this universality class.

It was pointed out forty years ago [1,2] that a superconducting state with an inhomogeneous order parameter can be stabilized by a Zeeman splitting between electrons with opposite spins, that is comparable to the energy gap. This inhomogeneous superconducting state, or Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, has been the subject of mostly theoretical study for many years [3]. The situation has changed recently as experimental results suggestive of the FFLO state start to emerge [4–11]. We would like to mention that recent experimental results on the heavy fermion compound $CeCoIn_5$, a quasi two-dimensional (2D) d-wave superconductor, are particularly compelling [9–11]. It is worth pointing out that the FFLO state may also be realized in high density quark matter, and is thus of interest to the particle physics community [3].

Given the long history of the subject matter, it is perhaps somewhat surprising that thus far most of the theoretical studies of the FFLO state are of the mean-field type. On the other hand one expects that quantum and thermal fluctuation effects should be much more significant in FFLO superconductors than in ordinary BCS superconductors, as the FFLO phase breaks the translational symmetry in addition to the gauge symmetry. In a recent work [12], one of us studied the FFLO phase in quasi 1D superconductors, and used bosonization to treat intra-chain quantum fluctuations exactly; one of the conclusions was that the transition from the FFLO phase to the BCS phase is a continuous transition of the commensurate-incommensurate type, in contrast to the first order transition commonly asserted in the literature. The effect of thermal fluctuations of the superconducting order parameter was discussed by Shimahara [13]; he argues that the enhanced fluctuation effects destabilize certain types of mean-field FFLO states. Anisotropy in pairing or electron dispersion may suppress these fluctuation effects however [13].

It is perhaps not quite well recognized yet that the FFLO superconductors are realizations of the Brazovskii model [14], which describes a large class of statistical mechanical systems in which the fluctuation spectra of the order parameter have their minima *away* from the origin in momentum space; this is precisely the case for the FFLO state, which prefers the superconducting order parameter to carry *finite* momenta. In its original form, the Brazovskii model assumes that the fluctuation spectrum has a continuous set of degenerate minima; it was shown that the fluctuation effects are so strong that they render the transition between the ordered and disordered phases a fluctuation-driven first order transition, even if the mean-field theory suggests a second-order transition. Interestingly, the transition between the normal and (possibly) FFLO phase in $CeCoIn_5$ was indeed found to be first-order [9,10]. There are two possible origins for the first order nature of the transition. Firstly, it is known that near the tricritical point where the normal, FFLO and BCS superconducting phases merge within mean field theory (at $T \approx 0.56 T_c$), the effective Ginzburg-Landau free energy has a contribution from quartic terms that is in certain cases *negative* [15,16], in which case the mean-field theory itself predicts a first-order transition. In $CeCoIn_5$ however, the FFLO phase was observed only at temperatures much lower than the tricritical point. It has been pointed out recently [17] that in the low-temperature regime the quartic term makes *positive* contributions to the free energy; thus a second-order transition would be expected at mean-field level. If this is the case then the origin of the first-order transition must be due to fluctuation effects left out at the mean-field level. It is this second possibility that we focus on in this work.

While the FFLO phase and the original Brazovskii model share the common feature that the order parameter spectrum has it minima away from the origin, one cannot apply the Brazovskii results directly to real systems like $CeCoIn_5$ because in such systems there is always anisotropy in either pairing potential or electron dispersion, which would generically reduce the continuous set of degenerate minima to a discrete set. Thus in this work we take this important effect into account, and use a model that contains the anisotropy that is appropriate for a d-wave superconductor with four-fold symmetry. We perform a renormalization group (RG) analysis

(combined with an appropriate ε expansion, see below), and show that the transition is generically first order (at least when ε is sufficiently small), even when the mean-field analysis suggests a second-order transition. In the following we first outline the derivation of an appropriate Ginzburg-Landau theory from a microscopic pairing model, then perform the RG analysis.

We consider a weak-coupling quasi 2D d-wave superconductor, whose partition function is $Z = \int D[\Psi^\dagger, \Psi] \exp\{-S\}$, with $S = S_0 + S_{int}$, where

$$S_0 = \sum_{\mathbf{k},\omega_n} \sum_{\sigma=\uparrow,\downarrow} [i\omega_n - \xi(\mathbf{k}) - \sigma I] \Psi_\sigma^\dagger(\mathbf{k}, \omega_n) \Psi_\sigma(\mathbf{k}, \omega_n),$$

$$S_{int} = -T \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \sum_{\nu,\nu',\omega} V_{\mathbf{k},\mathbf{k}'} \Psi_\uparrow^\dagger(\mathbf{k} + \mathbf{q}, \nu + \omega)$$

$$\cdot \Psi_\downarrow^\dagger(-\mathbf{k}, -\nu) \Psi_\uparrow(-\mathbf{k}', -\nu') \Psi_\downarrow(\mathbf{k}' + \mathbf{q}, \nu' + \omega), \quad (1)$$

with $\Psi, \Psi^\dagger$ being Grassman variables. Index $\sigma = \uparrow, \downarrow$ enumerates electron spin, and I is the Zeeman splitting that stabilizes the FFLO state [18]. The electron dispersion $\xi(\mathbf{k}) = \epsilon(\mathbf{k}_\parallel) + J \cos k_z d - \epsilon_F$, where $\mathbf{k}_\parallel$ denotes the momentum parallel to the planes, J is the strength of hopping between the layers. Interaction $V_{\mathbf{k},\mathbf{k}'} = V f_\mathbf{k} f'_\mathbf{k}$ with $V > 0$, is assumed for simplicity to be independent on the z-component of momenta with $f_\mathbf{k} = \cos 2\theta_\mathbf{k}$ for the d-wave pairing. We consider here the clean system only, assuming that disorder does not affect qualitatively the phenomena under consideration.

We decouple then the quartic term via the Hubbard-Stratanovich transformation by introducing the complex field $\Delta(\mathbf{q}, \omega_n)$, that serves as a superconducting order parameter. To obtain the Ginzburg-Landau (GL) action in powers of Δ, we perform subsequently the cumulant expansion integrating out the fermions having the Green function ($\sigma = \pm$ for up and down spins respectively) $G^{-1}(\mathbf{q}, \omega_n) = i\omega_n - \xi(\mathbf{q}) - \sigma I$. Considering here the finite-T transition only, we retain the zero-frequency component $\Delta(\mathbf{q}, \omega_n = 0)$ in all cumulants. The resulting functional takes the form

$$\mathcal{F} = \sum_\mathbf{q} K_2(\mathbf{q}) |\Delta(\mathbf{q})|^2$$

$$+ \sum_{\mathbf{q}_1,..,\mathbf{q}_4}' K_4(\mathbf{q}_1,..,\mathbf{q}_4) \Delta(\mathbf{q}_1) \Delta(\mathbf{q}_2) \Delta^*(\mathbf{q}_3) \Delta^*(\mathbf{q}_4), \quad (2)$$

where the prime in the sum over momenta in the quartic term indicates that the condition $\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{q}_3 - \mathbf{q}_4 = 0$ is taken into account. Since the transition occurs into the state that is non-uniform in space, the full momentum dependence of K_2 and K_4 must be kept [19]. $K_2(\mathbf{q})$ is given by the standard bubble diagram with two external legs, $K_2(\mathbf{q}) = 1/V - T \sum_{\mathbf{k},\nu_n} f_\mathbf{k}^2 G_\uparrow(\mathbf{k} + \mathbf{q}, \nu_n) \times G_\downarrow(-\mathbf{k}, -\nu_n)$. In this formula, one performs then straightforwardly the summation over frequency and integration over momentum within the shell around the Fermi surface $|k - k_F| \le \omega_D / v_F(\mathbf{k}_F)$. The form of the Fermi surface is assumed

to have the same 4-fold d-wave symmetry in the $k_\parallel$ plane. Distinguishing also the components of the Fermi-momentum parallel and perpendicular to the planes, $\mathbf{k}_F = (\mathbf{k}_\parallel^{(F)}, k_z^{(F)})$, $\mathbf{v}_F = (\mathbf{v}_\parallel, v_z)$, we find as a result that for $q \ll k_F$ [20]

$$K_2(\mathbf{q}) = \frac{1}{V} - \frac{1}{(2\pi)^3} \int_{-\pi/d}^{\pi/d} \frac{k_\parallel^{(F)} dk_z}{v_F(\mathbf{k}_F)} \int d\theta \cos^2 2\theta$$

$$\int_0^{\omega_D} \frac{d\epsilon}{2\epsilon} \left[\tanh\left(\frac{\epsilon + z_\mathbf{q}}{2T}\right) + \tanh\left(\frac{\epsilon - z_\mathbf{q}}{2T}\right) \right]. \quad (3)$$

In the equation above, $\hbar = 1$,

$$z_\mathbf{q} = \frac{1}{2} \left[v_\parallel q_\parallel \cos(\theta_\mathbf{v} - \theta_\mathbf{q}) - J d q_z \cdot \sin k_z d \right] + I, \quad (4)$$

with θ, $\theta_\mathbf{v}$ and $\theta_\mathbf{q}$ being the in-plane angles of $\mathbf{k}$, $\mathbf{v}_F$ and the pairing momentum $\mathbf{q}$ respectively. $\mathbf{k}_F$, $\mathbf{v}_F$ as well as $\theta_\mathbf{v}$, are themselves functions of k_z and θ characterizing the Fermi surface.

To determine the absolute value of the pairing momentum q_0 and its direction, it is necessary to find the minima of $K_2(\mathbf{q})$ with respect to $q_\parallel$ and $\theta_\mathbf{q}$, as well as q_z. It is clear that the ordering wave vector must lie in the (q_x, q_y) plane meaning that $q_{0z} = 0$. However, investigation of $K_2(\mathbf{q})$ regarding the minimum with respect to $\theta_\mathbf{q}$, reveals that Eq. (3) has extrema for the two sets of values: $\theta_i = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, -\frac{\pi}{4}$ and $\theta_i = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ correspond to the nodal and anti-nodal directions in the (q_x, q_y) plane. Generally speaking, for each of these sets one obtains the different values of q_0 as a result of the solution of equation $\partial K_2(\mathbf{q})/\partial q_0$. Whether both of these sets minimize $K_2(\mathbf{q})$, or only one of them is actually a minimum with the other being the maximum, is determined by the specific form of the Fermi surface. It is important that, if both sets with the corresponding values of q_0 are the minima, the actual transition will occur into configuration described by the set leading to the largest critical value T_c [20,21]. In case of the anti-nodal ordering, it is possible to expand $K_2(\mathbf{q})$ for $\mathbf{q}$ located in the pockets near the minima $q_{0x}^{(i)}, q_{0y}^{(i)}$, corresponding to the direction θ_i, $K_2(\mathbf{q}) = r + \alpha_x^{(i)}(q_x - q_{0x}^{(i)})^2 + \alpha_y^{(i)}(q_y - q_{0y}^{(i)})^2 + \gamma q_z^2$, where $\alpha_x^{(i)}$ and $\alpha_y^{(i)}$ mutually interchange for the neighboring pockets. For the case of nodal ordering, one can show from Eq. (3) that $K_2(\mathbf{q}) = r + \alpha(q_x - q_{0x}^{(i)})^2 + 2\beta^{(i)}(q_x - q_{0x}^{(i)})(q_y - q_{0y}^{(i)}) + \alpha(q_y - q_{0y}^{(i)})^2 + \gamma q_z^2$, where $\beta^{(i)}$ are opposite in sign for the neighboring pockets. By the simple rotation of coordinate system in the (q_x, q_y) plane by $\pi/4$, the latter expansion, however, reduces to that for the anti-nodal case.

The quartic kernel in Eq. (2) is given by the bubble containing four electron Green functions and four external legs representing the order parameter field $\Delta(\mathbf{q})$,

$$K_4 = (T/4) \sum_{\nu_n,\mathbf{k}} \left\{ f_\mathbf{k}^2 f_{\mathbf{k}+\mathbf{q}_4-\mathbf{q}_2}^2 G_\uparrow(\mathbf{k} + \mathbf{q}_1, \nu_n) G_\downarrow(-\mathbf{k}, -\nu_n) \right.$$

$$G_\uparrow(\mathbf{k} + \mathbf{q}_4, \nu_n) G_\downarrow(-\mathbf{k} + \mathbf{q}_2 - \mathbf{q}_4, -\nu_n) + [\mathbf{q}_4 \to \mathbf{q}_3]\}, \quad (5)$$

where $[\mathbf{q}_4 \to \mathbf{q}_3]$ stands for the same expression as right before, with only $\mathbf{q}_4$ substituted by $\mathbf{q}_3$. The kinematic constraint $\mathbf{q}_1 + \mathbf{q}_2 = \mathbf{q}_3 + \mathbf{q}_4$ is implied in Eq. (5). It will not be required, however, to know this cumulant for all values of momentum variables. Since we are considering the renormalization group treatment involving the wave-vectors located in the pockets near $\mathbf{q}_0^{(i)}$, only those values in Eq. (5) are of interest, in which $\mathbf{q}$'s point right to the centers of the pockets and satisfy the aforementioned constraint . The following distinct possibilities can be readily enumerated, once one denotes by i the number of the pocket in the $\mathbf{q}_\parallel$ plane; i equals to $1, 2, 3, 4$ starting from that with the lowest value of angle, with the formal condition $i + 4 = i$.

$$K_4(\mathbf{q}_0^{(i)}, \mathbf{q}_0^{(i)}, \mathbf{q}_0^{(i)}, \mathbf{q}_0^{(i)}) = u_0/4, i = 1, ..., 4; \quad (6)$$

$$K_4(\mathbf{q}_0^{(i)}, \mathbf{q}_0^{(i+1)}, \mathbf{q}_0^{(i)}, \mathbf{q}_0^{(i+1)}) = u_{\pi/2}/4, i = 1, ..., 4; \quad (7)$$

$$K_4(\mathbf{q}_0^{(i)}, \mathbf{q}_0^{(i+2)}, \mathbf{q}_0^{(i)}, \mathbf{q}_0^{(i+2)}) = u_\pi/4, \quad i = 1, 2; \quad (8)$$

$$K_4(\mathbf{q}_0^{(1)}, \mathbf{q}_0^{(3)}, \mathbf{q}_0^{(2)}, \mathbf{q}_0^{(4)}) = v/4. \quad (9)$$

Looking at the expressions given by Eqs. (6)-(9), we see that u_0 describes the interactions between the modes within the same pocket, while the other parameters account for the inter-pocket scattering. In parts with $u_{\pi/2}$ and u_π, the interaction occurs between the pockets that have the angles between their $\mathbf{q}_0$'s equal to $\pi/2$ and π respectively. Without providing the explicit expressions for those coefficients, we note that the only point important in the general derivation is that all interactions are non-singular at the critical values T_c and $\mathbf{q}_0$. The interactions can in principle have arbitrary signs that may change along the critical line $T_c = T_c(I)$ on the (T, I) plane. For example, as it was mentioned in the introduction, interaction u_0 is negative close to the tricritical point, where the normal, uniform and non-uniform superconducting phases meet [16]. At the same time, at lower temperatures u_0 seems to be positive [3]. If $u_0 < 0$, the transition is necessarily first order already at the mean-field level. Hence, we will assume that we consider the transition only in those regions on the phase diagram, in which at least u_0 is positive.

To distinguish the modes with the wave vectors belonging to the different pockets, it is convenient to introduce the shifted momenta $\mathbf{k} = \mathbf{q} - \mathbf{q}_0^{(i)}$ and decompose the total field $\Delta(\mathbf{q})$ into the parts $\Delta_i(\mathbf{k} = \mathbf{q} - \mathbf{q}_0^{(i)})$. Each part $\Delta_i(\mathbf{k})$ accounts for the fluctuations having the momenta in the vicinity of $\mathbf{q}_0^{(i)}$. It is clear that under such decomposition, the kinematic constraint $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 + \mathbf{k}_4$ for the shifted vectors is preserved. Though $\mathbf{k}_j$ in the arising

quartic terms generally belong to the different pockets, one can treat them during the formulation of RG equations, as if they are located in one and the same pocket around the origin. We will use in RG equations below the form of the propagator obtained for the anti-nodal ordering assuming for clarity that $\alpha_x^{(1)} = \alpha_1 \neq \alpha_y^{(1)} = \alpha_2$, meaning the spatial anisotropy in the spectrum of excitations. The issue of spatial anisotropy in RG near quantum critical points was addressed in different physical context in Ref. [22], albeit the anisotropy there was related to fermionic excitations.

After the appropriate rescaling of momentum variables and fields, the general GL action takes the form:

$$\mathcal{F} = \sum_i \sum_\mathbf{k} \left[r + \alpha_x^{(i)} k_x^2 + \alpha_y^{(i)} k_y^2 + k_z^2 \right] |\Delta_i(\mathbf{k})|^2$$

$$+ \sum_{\{k_j\}}' \left\{ (u_0/4) \sum_i \Delta_i(\mathbf{k}_1) \Delta_i(\mathbf{k}_2) \Delta_i^*(\mathbf{k}_3) \Delta_i^*(\mathbf{k}_4) \right.$$

$$+ u_{\pi/2} \sum_{[i]} \Delta_i(\mathbf{k}_1) \Delta_{i+1}(\mathbf{k}_2) \Delta_i^*(\mathbf{k}_3) \Delta_{i+1}^*(\mathbf{k}_4)$$

$$+ u_\pi \sum_{i=1,2} \Delta_i(\mathbf{k}_1) \Delta_{i+2}(\mathbf{k}_2) \Delta_i^*(\mathbf{k}_3) \Delta_{i+2}^*(\mathbf{k}_4)$$

$$+ v \left(\Delta_1(\mathbf{k}_1) \Delta_3(\mathbf{k}_2) \Delta_2^*(\mathbf{k}_3) \Delta_4^*(\mathbf{k}_4) + \text{c.c} \right\}. \quad (10)$$

A few more remarks on the notations in Eq. (10) are in order. The notation $\sum_{\{k_j\}}'$ implies that the summation over $\mathbf{k}_j$ is taken with the restriction $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3 + \mathbf{k}_4$. $\sum_{[i]}$ means that the sum over i is performed with the condition $i + 4 = i$. In all terms of the quartic part, except that with $u_0/4$, the permutational symmetry between the fields arising from the obvious permutations of momenta in arguments of Eqs. (7)-(9), is taken care of explicitly, canceling thus the factor of 4 in denominator. This greatly simplifies the subsequent RG loop analysis.

Couplings u_0 and u_π are in fact the primary parameters, whose flow under rescaling determines the character of transition. To see this, we calculate the free energy at the mean-field level for two possible phases: 1)Fulde-Ferell (FF) phase with $\Delta(\mathbf{r}) = \Delta_0 e^{i\mathbf{q}\mathbf{r}}$ and 2)Larkin-Ovchinnikov phase having $\Delta(\mathbf{r}) = \Delta_0 \cos(\mathbf{q} \cdot \mathbf{r})$. The values are $\mathcal{F}_{FF} = -|r|^2/u_0$, $\mathcal{F}_{LO} = -2|r|^2/(u_0 + 2u_\pi)$. The LO phase has the lower energy when $u_0 > 2u_\pi$, while the FF phase is more favorable under the opposite condition. The considerations above necessarily imply that $u_0 > 0$, since only in this case the transition is of the second order at the mean field level. In addition, if LO phase is realized, one must ensure that not only $u_0 > 0$ but also $u_0 + 2u_\pi > 0$. Those requirements will be presumed fulfilled in the subsequent treatment.

Simple tree-level scaling applied to Eq. (10) shows that if the effective dimensionality of the problem, $d > d_c = 4$, the interactions are irrelevant and the transition is of the second order. To proceed, we will generalize the z-component of momentum to $k_\perp$ having the dimen-

sionality $2 - \varepsilon$, and integrate out the modes in the thin layer $\Lambda/e^l < k_x, k_y < \Lambda$ around the square shell: $-\Lambda < k_x < \Lambda$, $k_y = \pm\Lambda$; $-\Lambda < k_y < \Lambda$, $k_x = \pm\Lambda$, with the integrals over $k_\perp$ taken from $-\infty$ to ∞. The arising one-loop RG equations for interactions are:

$$du_0/dl = \varepsilon u_0 - 2f\left[(5/4)u_0^2 + 2u_{\pi/2}^2 + u_\pi^2\right], \qquad (11)$$

$$du_\pi/dl = \varepsilon u_\pi - 2f\left[u_0 u_\pi + u_\pi^2 + u_{\pi/2}^2 + v^2/2\right], \quad (12)$$

$$\frac{du_{\pi/2}}{dl} = \varepsilon u_{\pi/2} - 2\left[f u_{\pi/2}(u_0 + u_\pi) + g(u_{\pi/2}^2 + \frac{v^2}{2})\right], \quad (13)$$

$$dv/dl = \varepsilon v - 2v\left[f u_\pi + 2g u_{\pi/2}\right], \qquad (14)$$

with

$$f = \frac{4}{(2\pi)^3}\int_{-\infty}^{\infty} k_\perp dk_\perp \left[\int_0^\Lambda dk_x \mathcal{G}^2(\alpha_1 k_x^2, \alpha_2\Lambda^2) \right.$$
$$\left. + \int_0^\Lambda dk_y \mathcal{G}^2(\alpha_1\Lambda^2, \alpha_2 k_y^2)\right],$$

$$g = \frac{4}{(2\pi)^3}\int_{-\infty}^{\infty} k_\perp dk_\perp \left[\int_0^\Lambda dk_x \mathcal{G}(\alpha_1 k_x^2, \alpha_2\Lambda^2)\cdot\right.$$
$$\left. \mathcal{G}(\alpha_2 k_x^2, \alpha_1\Lambda^2) + \int_0^\Lambda dk_y \mathcal{G}(\alpha_1\Lambda^2, \alpha_2 k_y^2)\mathcal{G}(\alpha_2\Lambda^2, \alpha_1 k_y^2)\right].$$

In the equations above, $\mathcal{G}(x, y) = 1/(k_\perp^2 + x + y)$, and we set $r(l) \sim O(\varepsilon)$ to zero in $\mathcal{G}(x, y)$ in the one-loop approximation. Looking for the fixed points, we absorb f by rescaling the interactions, generating thus the anisotropy parameter $a = g/f = (2\sqrt{\eta}/(\eta-1))\arcsin(\eta-1)/(\eta+1)$, where $\eta = \max\{\alpha_1/\alpha_2, \alpha_2/\alpha_1\} > 1$. This parameter is cutoff-independent and satisfies $0 < a \leq 1$. As it can be seen from Eq. (14), it is reasonable to search separately the fixed points that have $v^* = 0$ and $v^* \neq 0$. We have found that setting the condition $v^* \neq 0$, leads to the absence of fixed points in the space of real variables for $0 < a \leq 1$. Concerning the fixed points having $v^* = 0$, it can be shown that apart from the completely unstable Gaussian fixed point one has four more points: $u_{\pi/2}^* = u_\pi^* = 0, u_0^* = 2\varepsilon/5$; $u_{\pi/2}^* = 0, u_\pi^* = u_0^*/2 = \varepsilon/6$; plus two more fixed points that are some cumbersome functions of a not to be presented here. For $a = 1$, one easily finds the latter to be $u_{\pi/2}^* = u_\pi^* = u_0^*/2 = \varepsilon/8$, $u_{\pi/2}^* = u_\pi^* = u_0^*/6 = \varepsilon/16$; while for $a \to 0$ they both collapse onto the point $u_{\pi/2}^* = 0, u_\pi^* = u_0^*/2 = \varepsilon/6$. All of the found this way fixed points are unstable, since, as follows from Eq. (14), there will be at least one direction with the eigenvalue $\lambda_v = \varepsilon - 4u_{\pi/2}^* - 2u_\pi^*$, *positive* at all

the fixed points in the whole range $0 < a \leq 1$. We thus find no stable fixed points at the one-loop level, meaning that one needs to tune at least two parameters (r and v) to reach the fixed points. This implies that the transition will be generically of the first order, even if the mean-field theory suggests a second-order transition. The situation here is similar to that near transitions described by the effective Hamiltonians of anisotropic systems [23].

In summary, we have obtained an effective Ginsburg-Landau theory for the transition from normal to the FFLO state in quasi 2D d-wave superconductors at all non-zero temperatures. RG analysis of the theory indicates that the transition is generically first order, even when the mean-field theory suggest a continuous transition. This fluctuation-driven first order transition is due to the enhanced fluctuations of the FFLO state, associated with additional broken symmetries. Our result is consistent with the first order character of the transition observed in CeCoIn$_5$.

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[18] In real systems I can be due to either (i) an external magnetic field or (ii) an internal field if the system is ferromagnetic. In the case (i) there is also an orbital effect; however for the field that is parallel to the planes of a quasi 2D system the orbital effect is weak. In the following we neglect the orbital effect, so strictly speaking our results apply to case (ii) but should also be relevant to case (i) as well.

[19] The approach used here to obtain the effective GL action is different from previous work [15,16], which used a gradient expansion (assuming small order parameter momentum) that is justified near the tricritical point only; here we have kept the full momentum dependence of the kernels, which is essential to correctly identify the optimal momentum of the order parameter at low T.

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